Mathematical Approximation for Real-time Lighting Rendering Through Participating Media

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Abstract

Many shading models are able to provide realistic rendering of lighting effects under various atmospheric conditions but computational times are often expensive. This paper proposes a method to render lighting effects within participating media in real-time on a graphics workstation. It consists of mathematical approximations based on a re-formulation of the light transport equation considering atmospheric scattering with light sources described by their luminous intensity distribution. Hardware capabilities of graphics computer boards are used to accelerate parts of the rendering process.

1. Introduction

Real-time rendering has become a challenge for a wide panel of applications such as driving simulators, virtual-reality applications, industrial design... Algorithms based on 3D hardware allow realistic real-time rendering of surfaces lighting [5]. However, in the presence of participating media, few solutions have been proposed up today. Most realistic participating media rendering methods use finite elements [2][4][9] or Monte-Carlo techniques [3][8]. Another approach proposed by Nishita [7] uses a ray-tracing algorithm.

Previous methods are not suitable for real-time applications due to their expensive computational time. Real-time rendering solutions have been implemented in hardware but these latest considers only the exponential approximation of light attenuation due to the presence of media [6]. Other solutions take advantage of texture mapping techniques to reconstruct and view interactively a pre-computed volume grid via texture slices [1]. However, light sources are static in the medium.

In this paper, we propose a new approach for real time rendering of participating media based on the transport equation given by Nishita.

2. Mathematical approximation of light transport equation

We first consider the simplified transport equation given by Nishita [7] with classical notations:

$$L(0)=e^{-K_t d}L(S)+\frac{\Omega}{4\pi}\sum_{i=1}^{N}\int_0^d e^{-K_t u}.K_t.e^{-K_t r_i(u)}\frac{I_i(\vec{\omega}_i)}{r_i(u)^2}f(\alpha_i(u)).du$$

In this model, light sources are supposed to be punctual and the medium of constant density where the in-scattering phenomena is neglected. Radiance estimation is performed by considering a path along a ray decomposed into several path elements where light interaction within the medium is computed.

Rather than express the integral in terms of distance variation along the ray, we propose to express the integral in term of angular variation along the ray compared to the light source position.

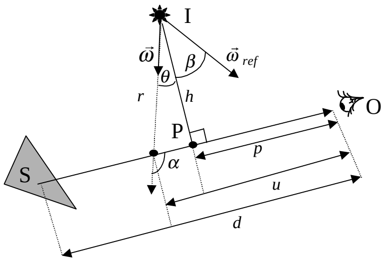

Figure 1: Geometry of the transport equation.

According to figure 1, the angular reformulation gives:

$$L(0)=L(S).e^{-K_t d}+K_t.\frac{\Omega}{4\pi}.\frac{e^{-K_t p}}{h}\int_{\theta_0}^{\theta_d} e^{-K_t h\left(\frac{\sin\theta+1}{\cos\theta}\right)}I(\theta+\beta)\,f\left(\theta+\frac{\pi}{2}\right)d\theta$$

Let consider now the following function, part of the integral term of the previous equation:

$$G(h,\theta,K_t)=e^{-K_t h\left(\frac{\sin\theta+1}{\cos\theta}\right)}f(\theta+\frac{\pi}{2})$$

For a given value of K_t and phase functions given by Nishita [7], this function only depends on two variables

θ and h, i.e. the relative positions of light and viewing direction. By applying a limited development in θ of *G*, the radiance equation becomes:

$$L(0) \approx L(S)e^{-K_t d} + K_t \frac{\Omega}{4\pi}\frac{e^{-K_t p}}{h}\begin{bmatrix} c_0(K_t,h)\int_{\theta_0}^{\theta_d} I(\theta+\beta)d\theta + \\ c_1(K_t,h)\int_{\theta_0}^{\theta_d} \theta I(\theta+\beta)d\theta + \ldots \end{bmatrix}$$

where c_0 and c_1 are constants depending only on the extinction coefficient K_t and the distance h.

3. Solutions for punctual and directional light sources for rendering method

3.1. Solution for a punctual light source

According to the previous polynomial approximation, a simple solution can be found for a homogenous punctual light source :

$$L(0) \approx L(S)e^{-K_t d} + K_t \frac{\Omega}{4\pi}\frac{e^{-K_t p}}{h}\left(\begin{array}{l} c_0(K_t,h)\left[\theta\right]_{\theta_0}^{\theta_d} + c_1(K_t,h)\left[\frac{\theta^2}{2}\right]_{\theta_0}^{\theta_d} \\ + c_2(K_t,h)\left[\frac{\theta^3}{3}\right]_{\theta_0}^{\theta_d} + \ldots \end{array} \right)$$

3.2. Solution for a directional light source

The evaluation of the equation for light sources with arbitrary intensity distribution is more complex. From the light source point of view, any ray (OS) belongs to a path P_i , i.e. an semicircle onto the hemisphere of intensity distribution as shown in figure 2:

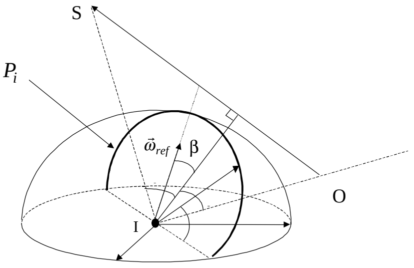

Figure 2 : A ray path onto the hemisphere.

The main idea is to sample, for each semicircle P_i, a set of M angles $\{\gamma_i, i = 1 \ldots M\}$ and to numerically pre-compute for each angle the following integral:

$$D(\gamma) = \int_{-\pi/2}^{\gamma} I(\theta+\beta)d\theta$$

then

$$\int_{\theta_0}^{\theta_d} I(\theta+\beta)d\theta = D(\gamma_d) - D(\gamma_0)$$

4. Implementation and results

During a first rendering pass, the radiance coming from surfaces is estimated using hardware accelerated fog function. In the same time, depth component of the image is extracted from the hardware Z-buffer. In a second additive pass, the medium contribution is computed into a texture which is apply onto the whole image using previous mathematical reformulation, where the depth component is used to determine the end point of the integral term. The mathematical reformulation and approximation of light transport equation allow us a very fast estimation of incoming luminance for each ray. Numerical integration and polynomial coefficients are pre-computed into tables for calculation efficiency.

5. Conclusion & perspectives

In this paper, we have proposed a method to render in a very fast way participating media considering light sources with arbitrary intensity distribution. This new rendering method can be applied in a real-time application using accelerated 3D hardware. In a next step, the lighting model should be improve in order to take into account the in-scattering phenomena.

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